

# Prediction of LSND effect as a "sterile" perturbation of the bimaximal texture for three active neutrinos\*

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## Abstract

As a contribution to the hypothesis of mixing of three active neutrinos with, at least, one sterile neutrino, we report on a simple  $4 \times 4$  texture whose  $3 \times 3$  part arises from the popular bimaximal texture for three active neutrinos  $\nu_e, \nu_\mu, \nu_\tau$ , where  $c_{12} = 1/\sqrt{2} = s_{12}$ ,  $c_{23} = 1/\sqrt{2} = s_{23}$  and  $s_{13} = 0$ . Such a  $3 \times 3$  bimaximal texture is perturbed through a rotation in the 14 plane, where  $\nu_4$  is the extra neutrino mass state induced by the sterile neutrino  $\nu_s$  which becomes responsible for the LSND effect. Then, with  $m_1^2 \simeq m_2^2$  we predict that  $\sin^2 2\theta_{\text{atm}} = \frac{1}{2}(1 + c_{14}^2) \sim 0.95$  and  $\sin^2 2\theta_{\text{LSND}} = \frac{1}{2}s_{14}^4 \sim 5 \times 10^{-3}$ , and in addition  $\Delta m_{\text{atm}}^2 = \Delta m_{32}^2$  and  $\Delta m_{\text{LSND}}^2 = |\Delta m_{41}^2|$ , where  $c_{14}^2 = \sin^2 2\theta_{\text{sol}} \sim 0.9$  and  $\Delta m_{21}^2 = \Delta m_{\text{sol}}^2 \sim 10^{-7} \text{ eV}^2$  if *e.g.* the LOW solar solution is applied.

PACS numbers: 12.15.Ff , 14.60.Pq , 12.15.Hh .

February 2001

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\*Supported in part by the Polish State Committee for Scientific Research (KBN).

The present status of experimental data for atmospheric  $\nu_\mu$ 's as well as solar  $\nu_e$ 's favours oscillations between three conventional neutrinos  $\nu_e, \nu_\mu, \nu_\tau$  only [1]. However, the problem of the third neutrino mass-square difference, related to the possible LSND effect for accelerator  $\nu_\mu$ 's, is still actual [2], stimulating a further discussion about mixing of these three active neutrinos with, at least, one hypothetical sterile neutrino  $\nu_s$  (although such a sterile neutrino is not necessarily able to explain the LSND effect [3]). As a contribution to this discussion, we report in this note on a simple  $4 \times 4$  texture for three active and one sterile neutrinos,  $\nu_e, \nu_\mu, \nu_\tau$  and  $\nu_s$ , whose  $3 \times 3$  part arises from the popular bimaximal texture [4] working *grossa modo* in a satisfactory way for solar  $\nu_e$ 's and atmospheric  $\nu_\mu$ 's if the LSND effect is ignored. Such a  $3 \times 3$  bimaximal texture is perturbed [5] by the sterile neutrino  $\nu_s$  inducing one extra neutrino mass state  $\nu_4$  and so, becoming responsible for the possible LSND effect. In fact, with the use of our  $4 \times 4$  texture we predict that  $\sin^2 2\theta_{\text{LSND}} = \frac{1}{2}s_{14}^4$  and  $\Delta m_{\text{LSND}}^2 = |\Delta m_{41}^2|$ , while  $\sin^2 2\theta_{\text{sol}} = c_{14}^2$  and  $\Delta m_{\text{sol}}^2 = \Delta m_{21}^2$  as well as  $\sin^2 2\theta_{\text{atm}} = \frac{1}{2}(1 + c_{14}^2)$  and  $\Delta m_{\text{atm}}^2 = \Delta m_{32}^2$ , if  $m_1^2 \simeq m_2^2$  (and both are different enough from  $m_3^2$  and  $m_4^2$ ). Here,  $c_{14}^2 \sim 0.9$  and  $\Delta m_{21}^2 \sim 10^{-7} \text{ eV}^2$  if *e.g.* the LOW solar solution [1] is accepted; then we predict  $\sin^2 2\theta_{\text{atm}} \sim 0.95$  and  $\sin^2 2\theta_{\text{LSND}} \sim 5 \times 10^{-3}$ .

In the popular  $3 \times 3$  bimaximal texture the mixing matrix has the form [4]

$$U^{(3)} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix}. \quad (1)$$

Such a form corresponds to  $c_{12} = 1/\sqrt{2} = s_{12}$ ,  $c_{23} = 1/\sqrt{2} = s_{23}$  and  $s_{13} = 0$  in the notation used for a generic Cabibbo–Kobayashi–Maskawa–type matrix [6] (if the LSND effect is ignored, the upper bound  $|s_{13}| \lesssim 0.1$  follows from the negative result of Chooz reactor experiment [7]). Going out from the form (1), we propose in the  $4 \times 4$  texture the following mixing matrix:

$$U = \begin{pmatrix} & & & 0 \\ & U^{(3)} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{14} & 0 & 0 & s_{14} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14} & 0 & 0 & c_{14} \end{pmatrix} = \begin{pmatrix} c_{14}/\sqrt{2} & 1/\sqrt{2} & 0 & s_{14}/\sqrt{2} \\ -c_{14}/2 & 1/2 & 1/\sqrt{2} & -s_{14}/2 \\ c_{14}/2 & -1/2 & 1/\sqrt{2} & s_{14}/2 \\ -s_{14} & 0 & 0 & c_{14} \end{pmatrix} \quad (2)$$

with  $c_{14} = \cos \theta_{14}$  and  $s_{14} = \sin \theta_{14}$  (note that in Eq. (2) only  $s_{12}$ ,  $s_{23}$  and  $s_{14}$  of all  $s_{ij}$

with  $i, j = 1, 2, 3, 4$ ,  $i < j$  are nonzero). The unitary transformation describing the mixing of four neutrinos  $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau, \nu_s$  is inverse to the form

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i, \quad (3)$$

where  $\nu_i = \nu_1, \nu_2, \nu_3, \nu_4$  denote four massive neutrino states carrying the masses  $m_i = m_1, m_2, m_3, m_4$ . Here,  $U = (U_{\alpha i})$ ,  $\alpha = e, \mu, \tau, s$  and  $i = 1, 2, 3, 4$ . Of course,  $U^\dagger = U^{-1}$  and also  $U^* = U$ , so that a tiny CP violation is ignored.

In the representation, where the mass matrix of three charged leptons  $e^-, \mu^-, \tau^-$  is diagonal, the  $4 \times 4$  neutrino mixing matrix  $U$  is at the same time the diagonalizing matrix for the  $4 \times 4$  neutrino mass matrix  $M = (M_{\alpha\beta})$ :

$$U^\dagger M U = \text{diag}(m_1, m_2, m_3, m_4), \quad (4)$$

where by definition  $m_1^2 \leq m_2^2 \leq m_3^2$  and *either*  $m_4^2 \leq m_1^2$  *or*  $m_3^2 \leq m_4^2$ . Then, due to the formula  $M_{\alpha\beta} = \sum_i U_{\alpha i} m_i U_{\beta i}^*$  we obtain

$$\begin{aligned} M_{ee} &= \frac{1}{2} (c_{14}^2 m_1 + s_{14}^2 m_4 + m_2), \\ M_{e\mu} &= -M_{e\tau} = -\frac{1}{2\sqrt{2}} (c_{14}^2 m_1 + s_{14}^2 m_4 - m_2), \\ M_{\mu\mu} &= M_{\tau\tau} = \frac{1}{2} \left[ \frac{1}{2} (c_{14}^2 m_1 + s_{14}^2 m_4 + m_2) + m_3 \right] = M_{ee} + M_{\mu\tau}, \\ M_{\mu\tau} &= -\frac{1}{2} \left[ \frac{1}{2} (c_{14}^2 m_1 + s_{14}^2 m_4 + m_2) - m_3 \right], \\ M_{es} &= -\frac{1}{\sqrt{2}} c_{14} s_{14} (m_1 - m_4), \\ M_{\mu s} &= -M_{\tau s} = \frac{1}{2} c_{14} s_{14} (m_1 - m_4) = -\frac{1}{\sqrt{2}} M_{es}, \\ M_{ss} &= s_{14}^2 m_1 + c_{14}^2 m_4. \end{aligned} \quad (5)$$

Of course,  $M^\dagger = M^{-1}$  and also  $M^* = M$ . From Eqs. (5) we find that

$$\begin{aligned} m_{1,4} \text{ or } m_{4,1} &= \frac{M_{ee} - M_{e\mu}\sqrt{2} + M_{ss}}{2} \pm \sqrt{\left( \frac{M_{ee} - M_{e\mu}\sqrt{2} - M_{ss}}{2} \right)^2 + 2M_{es}^2}, \\ m_2 &= M_{ee} + M_{e\mu}\sqrt{2}, \quad m_3 = M_{\mu\mu} + M_{\mu\tau} \end{aligned} \quad (6)$$

if  $m_4 \leq m_1$  or  $m_1 \leq m_4$ , respectively, and

$$(2c_{14}s_{14})^2 = \frac{8M_{es}^2}{(M_{ee} - M_{e\mu}\sqrt{2} - M_{ss})^2 + 8M_{es}^2} . \quad (7)$$

Obviously,  $m_1 + m_2 + m_3 + m_4 = M_{ee} + M_{\mu\mu} + M_{\tau\tau} + M_{ss}$  as  $M_{ee} = M_{\mu\mu} - M_{\mu\tau}$  and  $M_{\mu\mu} = M_{\tau\tau}$ .

Due to the mixing of four neutrino fields described in Eq. (3), neutrino states mix according to the form

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle . \quad (8)$$

This implies the following familiar formulae for probabilities of neutrino oscillations  $\nu_\alpha \rightarrow \nu_\beta$  on the energy shell:

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\langle \nu_\beta | e^{iPL} | \nu_\alpha \rangle|^2 = \delta_{\beta\alpha} - 4 \sum_{j>i} U_{\beta j}^* U_{\beta i} U_{\alpha j} U_{\alpha i}^* \sin^2 x_{ji} , \quad (9)$$

valid if the quartic product  $U_{\beta j}^* U_{\beta i} U_{\alpha j} U_{\alpha i}^*$  is real, what is certainly true when a tiny CP violation is ignored (then  $U_{\alpha i}^* = U_{\alpha i}$ ). Here,

$$x_{ji} = 1.27 \frac{\Delta m_{ji}^2 L}{E} , \quad \Delta m_{ji}^2 = m_j^2 - m_i^2 \quad (10)$$

with  $\Delta m_{ji}^2$ ,  $L$  and  $E$  measured in  $\text{eV}^2$ , km and GeV, respectively ( $L$  and  $E$  denote the experimental baseline and neutrino energy, while  $p_i = \sqrt{E^2 - m_i^2} \simeq E - m_i^2/2E$  are eigenstates of the neutrino momentum  $P$ ).

With the use of oscillation formulae (9), the proposal (2) for the  $4 \times 4$  neutrino mixing matrix leads to the probabilities

$$\begin{aligned} P(\nu_e \rightarrow \nu_e) &\simeq 1 - c_{14}^2 \sin^2 x_{21} - (1 + c_{14}^2) s_{14}^2 \sin^2 x_{41} , \\ P(\nu_\mu \rightarrow \nu_\mu) &= P(\nu_\tau \rightarrow \nu_\tau) \simeq 1 - \frac{1}{4} c_{14}^2 \sin^2 x_{21} - \frac{1}{2} (1 + c_{14}^2) \left( \sin^2 x_{32} + \frac{1}{2} s_{14}^2 \sin^2 x_{41} \right) \\ &\quad - \frac{1}{2} s_{14}^2 \sin^2 x_{43} , \\ P(\nu_\mu \rightarrow \nu_e) &= P(\nu_\tau \rightarrow \nu_e) \simeq \frac{1}{2} \left( c_{14}^2 \sin^2 x_{21} + s_{14}^4 \sin^2 x_{41} \right) , \\ P(\nu_\mu \rightarrow \nu_\tau) &\simeq -\frac{1}{4} c_{14}^2 \sin^2 x_{21} + \frac{1}{2} (1 + c_{14}^2) \left( \sin^2 x_{32} - \frac{1}{2} s_{14}^2 \sin^2 x_{41} \right) \\ &\quad + \frac{1}{2} s_{14}^2 \sin^2 x_{43} \end{aligned} \quad (11)$$

in the approximation, where  $m_1^2 \simeq m_2^2$  (and both are different enough from  $m_3^2$  and  $m_4^2$ ). The probabilities involving the sterile neutrino  $\nu_s$  read:

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_s) &= P(\nu_\tau \rightarrow \nu_s) = (c_{14}s_{14})^2 \sin^2 x_{41} , \\ P(\nu_e \rightarrow \nu_s) &= 2(c_{14}s_{14})^2 \sin^2 x_{41} , \\ P(\nu_s \rightarrow \nu_s) &= 1 - (2c_{14}s_{14})^2 \sin^2 x_{41} . \end{aligned} \quad (12)$$

If  $\Delta m_{21}^2 \ll |\Delta m_{41}^2|$  (*i.e.*,  $x_{21} \ll |x_{41}|$ ) and

$$\Delta m_{21}^2 = \Delta m_{\text{sol}}^2 \sim 10^{-7} \text{ eV}^2 , \quad (13)$$

then, under the conditions of solar experiments the first Eq. (11) gives

$$P(\nu_e \rightarrow \nu_e)_{\text{sol}} \simeq 1 - c_{14}^2 \sin^2(x_{21})_{\text{sol}} - \frac{1}{2}(1 + c_{14}^2)s_{14}^2 \quad (14)$$

with the estimate

$$c_{14}^2 = \sin^2 2\theta_{\text{sol}} \sim 0.9 , \quad \frac{1}{2}(1 + c_{14}^2)s_{14}^2 \sim 0.095 . \quad (15)$$

In Eqs. (13) and (15) the LOW solar solution [1,8] is used. Note that

$$P(\nu_e \rightarrow \nu_e)_{\text{sol}} \simeq 1 - P(\nu_e \rightarrow \nu_\mu)_{\text{sol}} - P(\nu_e \rightarrow \nu_\tau)_{\text{sol}} - (c_{14}s_{14})^2 \quad (16)$$

with  $(c_{14}s_{14})^2 \sim 0.09$ .

If  $\Delta m_{21}^2 \ll \Delta m_{32}^2 \ll |\Delta m_{41}^2|$ ,  $|\Delta m_{43}^2|$  (*i.e.*,  $x_{21} \ll x_{32} \ll |x_{41}|$ ,  $|x_{43}|$ ) and

$$\Delta m_{32}^2 = \Delta m_{\text{atm}}^2 \sim 3 \times 10^{-3} \text{ eV}^2 , \quad (17)$$

then for atmospheric experiments the second Eq. (11) leads to

$$P(\nu_\mu \rightarrow \nu_\mu)_{\text{atm}} \simeq 1 - \frac{1}{2}(1 + c_{14}^2) \sin^2(x_{32})_{\text{atm}} - \frac{1}{8}(3 + c_{14}^2)s_{14}^2 \quad (18)$$

with the prediction

$$\sin^2 2\theta_{\text{atm}} = \frac{1}{2}(1 + c_{14}^2) \sim 0.95 , \quad \frac{1}{8}(3 + c_{14}^2)s_{14}^2 \sim 0.049 \quad (19)$$

following from the value (15) of  $c_{14}^2$ . Notice that

$$P(\nu_\mu \rightarrow \nu_\mu)_{\text{atm}} \simeq 1 - P(\nu_\mu \rightarrow \nu_\tau)_{\text{atm}} - \frac{1}{4}(1 + c_{14}^2)s_{14}^2 \quad (20)$$

with  $(1 + c_{14}^2)s_{14}^2/4 \sim 0.048$ .

Eventually, if  $\Delta m_{21}^2 \ll |\Delta m_{41}^2|$  (*i.e.*,  $x_{21} \ll |x_{41}|$ ) and

$$|\Delta m_{41}^2| = \Delta m_{\text{LSND}}^2 \sim 1 \text{ eV}^2, \quad (21)$$

then for the LSND accelerator experiment the third Eq. (11) implies

$$P(\nu_\mu \rightarrow \nu_e)_{\text{LSND}} \simeq \frac{1}{2}s_{14}^4 \sin^2(x_{41})_{\text{LSND}} \quad (22)$$

with the prediction

$$\sin^2 2\theta_{\text{LSND}} = \frac{1}{2}s_{14}^4 \sim 5 \times 10^{-3} \quad (23)$$

inferred from the value (15) of  $c_{14}^2$ . Such a prediction for  $\sin^2 2\theta_{\text{LSND}}$  is not inconsistent with the estimate  $\Delta m_{\text{LSND}}^2 \sim 1 \text{ eV}^2$  [2]. Note that

$$P(\nu_\mu \rightarrow \nu_e)_{\text{LSND}} \simeq \frac{1}{2} \left( \frac{s_{14}}{c_{14}} \right)^2 P(\nu_\mu \rightarrow \nu_s)_{\text{LSND}} \quad (24)$$

with  $\frac{1}{2}(s_{14}/c_{14})^2 \sim 0.062$ .

Concluding, we can say that Eqs. (14), (18) and (22) are consistent with solar, atmospheric and LSND experiments. All three depend on one common correlating parameter  $c_{14}^2$ , implying  $c_{14}^2 = \sin^2 2\theta_{\text{sol}} \sim 0.9$ ,  $\sin^2 2\theta_{\text{atm}} = \frac{1}{2}(1 + c_{14}^2) \sim 0.95$  and  $\sin^2 2\theta_{\text{LSND}} = \frac{1}{2}s_{14}^4 \sim 5 \times 10^{-3}$ . They depend also on three different mass-square scales  $\Delta m_{21}^2 = \Delta m_{\text{sol}}^2 \sim 10^{-7} \text{ eV}^2$ ,  $\Delta m_{32}^2 = \Delta m_{\text{atm}}^2 \sim 3 \times 10^{-3} \text{ eV}^2$  and  $|\Delta m_{41}^2| = \Delta m_{\text{LSND}}^2 \sim 1 \text{ eV}^2$ . Here, the LOW solar solution [1,8] is accepted. Note that in Eqs. (14) and (18) there are constant terms which modify moderately the usual two-flavor formulae. Any LSND-type accelerator project, in contrast to the solar and atmospheric experiments, investigates a small  $\nu_\mu \rightarrow \nu_e$  oscillation effect caused possibly by the sterile neutrino  $\nu_s$ . Thus, this effect (if it exists) plays the role of a small "sterile" perturbation of the basic bimaximal texture for three active neutrinos  $\nu_e, \nu_\mu, \nu_\tau$ . Of course, if  $s_{14}$  were zero, the LSND effect would not exist and both solar  $\nu_e \rightarrow \nu_e$  and atmospheric  $\nu_\mu \rightarrow \nu_\mu$  oscillations would be maximal. So,

from the standpoint of our texture (2), the estimated not full maximality of solar  $\nu_e \rightarrow \nu_e$  oscillations may be considered as an argument for the existence of the LSND effect.

The final results (14), (18) and (22) follow from the first three oscillation formulae (11), if *either*

$$m_4^2 \ll m_1^2 \simeq m_2^2 \simeq m_3^2 \quad (25)$$

with

$$m_1^2 \sim 1 \text{ eV}^2, \quad m_4^2 \ll 1 \text{ eV}^2, \quad \Delta m_{21}^2 \sim 10^{-7} \text{ eV}^2, \quad \Delta m_{32}^2 \sim 3 \times 10^{-3} \text{ eV}^2 \quad (26)$$

or

$$m_1^2 \simeq m_2^2 \ll m_3^2 \ll m_4^2 \quad (27)$$

with

$$m_1^2 \ll 1 \text{ eV}^2, \quad m_4^2 \sim 1 \text{ eV}^2, \quad \Delta m_{21}^2 \sim 10^{-7} \text{ eV}^2, \quad \Delta m_{32}^2 \sim 3 \times 10^{-3} \text{ eV}^2. \quad (28)$$

In both cases  $\Delta m_{21}^2 \ll \Delta m_{32}^2 \ll |\Delta m_{41}^2| \sim 1 \text{ eV}^2$ . The first case of  $m_4^2 \ll m_1^2 \sim 1 \text{ eV}^2$ , where the neutrino mass state  $\nu_4$  induced by the sterile neutrino  $\nu_s$  gets a vanishing mass, seems to be more natural than the second case of  $m_3^2 \ll m_4^2 \sim 1 \text{ eV}^2$ , where such a state gains a considerable amount of mass  $\sim 1 \text{ eV}$  "for nothing". This is so, unless one believes in the liberal maxim "whatever is not forbidden is allowed". Note that in the first case the neutrino mass states  $\nu_1, \nu_2, \nu_3$  get their considerable masses  $\sim 1 \text{ eV}$  through spontaneously breaking the electroweak  $\text{SU}(2)_L \times \text{U}(1)$  symmetry which, if it were not broken, would forbid these masses.

Finally, for the Chooz reactor experiment [5], where it happens that  $(x_{ji})_{\text{Chooz}} \simeq (x_{ji})_{\text{atm}}$ , the first Eq. (11) predicts

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e)_{\text{Chooz}} \simeq P(\bar{\nu}_e \rightarrow \bar{\nu}_e)_{\text{atm}} \simeq 1 - \frac{1}{2}(1 + c_{14}^2)s_{14}^2 \quad (29)$$

with  $\frac{1}{2}(1 + c_{14}^2)s_{14}^2 \sim 0.095$ . In terms of the usual two-flavor formula, the negative result of Chooz experiment excludes the disappearance process of reactor  $\bar{\nu}_e$ 's for moving

$\sin^2 2\theta_{\text{Chooz}} \gtrsim 0.1$  and  $\Delta m_{\text{Chooz}}^2 \gtrsim 3 \times 10^{-3} \text{ eV}^2$ . In our case  $\sin^2 2\theta_{\text{Chooz}} = \frac{1}{2}(1 + c_{14}^2)s_{14}^2 \sim 0.095$  for  $\sin^2 x_{\text{Chooz}} \sim 1$ . Thus, the Chooz effect for reactor  $\bar{\nu}_e$ 's may appear at the edge (if only the LSND effect exists with  $\sin^2 2\theta_{\text{LSND}} = \frac{1}{2}s_{14}^4 \sim 5 \times 10^{-3}$ ).

From the neutrinoless double  $\beta$  decay, not observed so far, the experimental bound  $\overline{M}_{ee} \equiv |\sum_i U_{ei}^2 m_i| \lesssim [0.4 \text{ (0.2)} - 1.0(0.6)] \text{ eV}$  follows [9] (here,  $U_{ei}^2$  appears even if  $U_{ei}^* \neq U_{ei}$ ). On the other hand, with the values  $c_{14}^2 \sim 0.9$  and  $s_{14}^2 \sim 0.1$  the first Eq. (5) gives

$$\overline{M}_{ee} = |M_{ee}| \sim \frac{1}{2}|0.9m_1 + 0.1m_4 + m_2|, \quad (30)$$

what in the case of Eq. (25) with  $m_1 \sim \pm 1 \text{ eV}$  and  $m_2 \sim 1 \text{ eV}$  or Eq. (27) with  $|m_4| \sim 1 \text{ eV}$  leads to the estimation  $\overline{M}_{ee} \sim (0.95, 0.05) \text{ eV}$  or  $\overline{M}_{ee} \sim 0, 05 \text{ eV}$ , respectively (putting  $\overline{M}_{ee} = |M_{ee}|$  in Eq. (30) we ignore a tiny — as we believe — violation of CP: we get  $U_{ei}^* = U_{ei}$ , since  $M_{ee} = \sum_i |U_{ei}|^2 m_i$ ).

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